The foregoing equations are valid locally and instantaneously, i.e., at a given particle X and time t, no matter what the conditions may be at surrounding particles or previous times. The conditions need not be, and usually are not, uniformly valid in the sense that any quantity appearing in them remains constant. Numerous discussions of the growth or decay of waves have been given.[9,10]

IV. ACCELERATION WAVES

In this section, attention will be directed to the analysis of acceleration waves. First, the consequences of the conditions of definition of an acceleration wave are examined. Since $U = u_n - \dot{x}_n$ and $[u_n] = 0$, then $[U] = -[\dot{x}_n]$. Since $[\dot{x}^k] = 0$ for an acceleration wave, it follows that $[\dot{x}_n] = 0$ and

$$[U] = 0. \tag{4.1}$$

It is immediately clear that by (3.3)

$$[\rho] = 0. \tag{4.2}$$

By (4.1), (4.2), and the definition of an acceleration wave, (3.4) takes the form

$$[t^{km}]n_m = 0 \tag{4.3}$$

for an acceleration wave. Now, consider the Piola-Kirchhoff stress T^{kK} which is related to the Cauchy stress t^{km} by the equation[11]

$$t^{km} n_m da = T^{kK} N_K dA, \tag{4.4}$$

where da is the area element in the deformed material and dA is the area element in the undeformed material. Here, the relation between n and N is given by Eq. (182.8) of Ref. 8. By (4.3) and (4.4) it follows immediately that

$$[T^k] = [T^{kK}]N_K = 0, \tag{4.5}$$

where $T^k = T^{kK}N_K$. Therefore, by requiring relations (3.3) and (3.4) be satisfied, it follows that the density and the stress vector must be continuous across an acceleration wave.

The jumps in the second derivatives of motion are given by

$$[\ddot{x}^k] = U^2 a^k, \qquad [\dot{x}^k_{,m}] = -U a^k n_m, \tag{4.6}$$

where a^k are the components of an arbitrary surface vector called the wave amplitude. It follows immediately that

$$[\dot{x}^k_{,K}]U = -[\ddot{x}^k]n_k. \tag{4.7}$$

Since the differential form of the conservation of mass must be satisfied on either side of a singular surface, then

$$[\dot{\rho} + \rho \dot{x}^k_{,K}] = 0, \tag{4.8}$$

which reduces to

$$[\dot{\rho}] + \rho[\dot{x}^k_{,K}] = 0 \tag{4.9}$$

for an acceleration wave. Furthermore, (4.9) with (4.7) becomes

$$[\dot{\rho}]U - \rho[\ddot{x}^k]n_k = 0. \tag{4.10}$$

Since $[T^k] = 0$, and applying the dual of Eq. (180.4) of Ref. 8 to T^k, it follows that

$$[\dot{T}^k] = -U_N[N^L T^k_{,L}] = -U_N[N^L T^{kK}_{,L} N_K + T^{kK} N^L N_{K,L}]. \tag{4.11}$$

In addition, if $[T^{kK}] = 0$, it can be shown that (4.11) together with the differential form of the conservation of linear momentum reduces to

$$[\dot{T}^{kK}]N_K = -\rho_0 U_N [\ddot{x}^k], \tag{4.12}$$

where ρ_0 is the density at $t = t_0$. Furthermore, it can be shown without any additional assumptions that (4.12) has the alternate form

$$[\dot{t}^{km}]n_m = -\rho U[\ddot{x}^k]. \tag{4.13}$$

Thus, two very important relations, given by (4.10) and (4.13), have been derived. It is clear that these conditions are arrived at independent of the material in question. Therefore, they may be used to determine the rate of change of density and the rate of change of the stress behind an acceleration wave. On the other hand, (4.13) may be used to determine the local speed of propagation of an acceleration wave in a particular material when its constitutive relation is given.

V. SPECIALIZATION TO ONE-DIMENSIONAL MOTIONS

The relations (4.10) and (4.13) are, of course, quite general and must be specialized for application to the analysis of plane longitudinal one-dimensional motions. Let the x^1 axis be coincident with such a motion. Then $n^1 = 1$ and $\ddot{x}^1 \equiv \ddot{x}$ are the only nonzero components of the normal and the acceleration. In this situation (4.10) and (4.13) imply that

$$[\dot{\rho}]U - \rho[\ddot{x}] = 0, \quad \text{and} \quad [\dot{\sigma}] + \rho U[\ddot{x}] = 0, \tag{5.1}$$

where $\sigma \equiv t^{11}$. The strain ϵ is given by

$$\epsilon = (\rho_0/\rho) - 1.$$

Consequently,

$$\dot{\rho} = -(\rho^2/\rho_0)\dot{\epsilon};$$

and since $\rho U = \rho_0 U_N$, the relations (5.1) may be rewritten in the forms

$$[\dot{\epsilon}] = -(1/U_N)[\ddot{x}], \qquad [\dot{\sigma}] = -\rho_0 U_N [\ddot{x}], \tag{5.2}$$

which are most convenient for the analysis of particle-velocity–time data. When stress-time data are to be reduced, the forms

$$[\dot{\epsilon}] = (1/\rho_0 U_N^2)[\dot{\sigma}], \qquad [\ddot{x}] = -(1/\rho_0 U_N)[\dot{\sigma}] \tag{5.3}$$

are appropriate. The quantity ρ_0, the density of the body in the reference state, is a constant known in advance. The use of the measure U_N of the wave speed is also

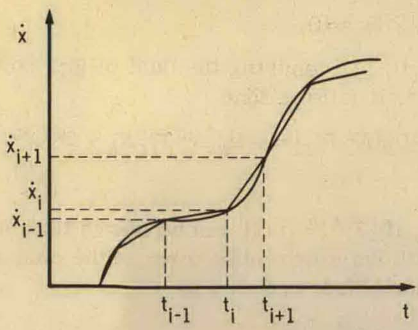

FIG. 1. Recorded velocity history with an approximation by a sequence of chords. The discontinuities where the chords join represent acceleration waves.

convenient, as it is the quantity inferred by measurement of wave transit time between given fixed material particles.

To illustrate the application of Eq. (5.1) to the interpretation of experimental data the particle-velocity history shown in Fig. 1 is considered as an example. This history is considered to have been recorded at a given plane in the interior of a sample without interference with the propagating wave. The actual recorded waveform is approximated by a system of chords such as that shown. Further attention is restricted to this piecewise linear approximation. The discontinuities in slope where the chords meet are acceleration waves, and are treated as outlined in the preceding paragraphs.

If, for simplicity, the chord approximation is chosen so that the arrivals of the acceleration waves at the recording station are all equally spaced in time by an amount Δt, the particle accelerations in the regions ahead of an behind the ith wave are

$$\ddot{x}_i^+ = (\dot{x}_i - \dot{x}_{i-1})/\Delta t, \quad \ddot{x}_i^- = (\dot{x}_{i+1} - \dot{x}_i)/\Delta t,$$

respectively, and we have

$$[\ddot{x}]_i = -(\dot{x}_{i-1} - 2\dot{x}_i + \dot{x}_{i+1})/\Delta t. \tag{5.4}$$

Similar relations hold for other quantities of interest of interest although they will be only approximately correct, even if the actual particle-velocity record is piecewise linear. Relations corresponding to (5.4) for jumps in stress and strain rates are

$$\begin{aligned} [\dot{\sigma}]_i &= -(\sigma_{i-1} - 2\sigma_i + \sigma_{i+1})/\Delta t, \\ [\dot{\epsilon}]_i &= -(\epsilon_{i-1} - 2\epsilon_i + \epsilon_{i+1})/\Delta t. \end{aligned} \tag{5.5}$$

Substitution of (5.4) and (5.5) into (5.2) gives the formulas

$$\begin{aligned} \epsilon_{i+1} &= 2\epsilon_i - \epsilon_{i-1} - (1/U_{Ni})(\dot{x}_{i-1} - 2\dot{x}_i + \dot{x}_{i+1}), \\ \sigma_{i+1} &= 2\sigma_i - \sigma_{i-1} - \rho_0 U_{Ni}(\dot{x}_{i-1} - 2\dot{x}_i + \dot{x}_{i+1}), \end{aligned} \tag{5.6}$$

from which the stress and strain at the $(i+1)$st wave can be calculated from the stress and strain at previous waves, recorded values of particle velocity, and a knowledge of the propagation velocity of the ith wave. A similar calculation based on (5.3) yields formulas for finding ϵ and $\dot{x}$ from measured stress histories.

It should be pointed out that the quantity $U_{Ni}(t)$ appearing in (5.6) is the acceleration wave speed which, for very large classes of materials, is the sound speed.[12,13] In order to carry out data reduction with (5.6), this speed must be known in advance or determined experimentally. First, if the velocity-time data or the stress-time data are available for two stations, then the instantaneous wave speed may be approximated from calculations based on the difference in arrival times of each particle-velocity or stress level. On the other hand, if both the velocity-time data and the stress-time data are available at a single station, then the wave speed follows directly by applying (5.6).

Finally, it should be pointed out that many experiments involve perturbations to the incident wave shapes. As demonstrated in the Appendix, the acceleration wave theory provides a basis for solving the resulting wave interaction problems.

VI. DISCUSSION

The acceleration wave theory presented here and the theory of Fowles and Williams[7] are broadly applicable to many solids because these theories involve no assumptions concerning the constitutive relation. However, the two theories lead to significantly different demands upon the experimental determination of material response from wave-propagation experiments. The dual-wave theory requires measurements of both stress and velocity histories. On the other hand, the acceleration wave analysis requires only measurement of either the stress or velocity history. Both theories require measurements of wave speed. In the acceleration wave theory, the average speed of either the stress or the velocity wave between two closely spaced stations is employed as an approximation to the instantaneous sound speed, whereas the dual-wave theory requires simultaneous measurements of both stress and velocity waves. According to the acceleration wave theory, simultaneous measurements of stress and particle velocity at a single station permits the instantaneous acceleration wave speed to be calculated.

In effect, the acceleration wave theory places no new demands upon the experiments beyond those customary in shock-wave studies, even though the theory is applicable to more complex material response. Since previous analyses of waves are known to provide a close approximation to the real response, the acceleration wave theory should provide an adequate base for analyzing the response of many solids with the use of existing experimental instruments and techniques. Furthermore, as shown in the Appendix, the acceleration wave theory leads to fairly simple and straightforward techniques for solving wave interaction problems which are frequently introduced by the measuring instruments.

In contrast to the acceleration wave theory, the dual-wave theory appears to require experiments which are not possible with existing capabilities. Furthermore, if that capability were developed it is not clear whether it would result in any significant change in the end result of describing material response. Butcher[14] has performed computer analyses of several rate-dependent solids which indicate that differences in wave speeds are too small to be significant.